2025

Four Year Under Graduate Programme (FYUGP)

2nd Semester Examination (Under NEP)
(New Session 2024-25)

MATHEMATICS (Major)

Paper Code : MTMMJ DC-201

(Calculus and Analytical Geometry)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Answer question no. 1 and any *four* from the rest.

1. Answer any *five* questions : $2\times5=10$

(a) Show that $\lim\limits_{x\to 0} \sin\dfrac{1}{x}$ does not exist.

(b) Give an example of a real-valued function which has a removable discontinuity.

(c) Find the nature of the conic

$$\frac{1}{r}=8+5\cos\theta.$$

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(d) Find the radius of curvature of the curve $y = x e^{-x}$ at the point where y attains its maximum.

(e) Prove that

$$\int_0^{\pi/2} \sin^P x\,dx \times \int_0^{\pi/2} \sin^{P+1} x\,dx = \frac{\pi}{2(P+1)}.$$

(f) Find the equation of a right circular cone which passes through the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and whose axis is $\frac{x}{1}=\frac{y}{-2}=\frac{z}{2}$.

(g) Test whether the equation $x^2+6xy+9y^2-5x-15y+6=0$ represents a pair of parallel straight lines.

(h) Find the value of $B\left(\frac{1}{2},\frac{1}{2}\right)$.

2. (a) If $y=e^{m\sin^{-1}x}$, prove that

$$\left(1-x^2\right)y_{n+2}-(2n+1)xy_{n+1}-\left(n^2+m^2\right)y_n=0.$$

Hence, find y_n at $x=0$. 4+1

(b) If $I_n=\int_0^{\pi/2} x^n \sin x\,dx$, n being a positive integer greater than 1, show that

$I_n+n(n-1)I_{n-2}=n\left(\frac{\pi}{2}\right)^{n-1}$. Hence find the value of $\int_0^{\pi/2} x^5 \sin x\,dx$. 4+1

3. (a) Find the range of values of x for which $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upwards or downwards. Find also its point/points of inflexion, if any. 2+2+1

(b) Find the envelope of the parabola $\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1$, where $ab = c^2$, c is a fixed constant, a and b being variable parameters. 3

(c) Find the value of $\int_0^\infty e^{-x^2} dx$ and hence find the value of $\int_{-\infty}^\infty e^{-x^2} dx$. 2

4. (a) If the straight line $lx + my + n = 0$ is a normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, prove that

$$\frac{a^2}{b^2} - \frac{l^2}{m^2} = \frac{\left(a^2 + b^2\right)^2}{n^2}.$$ 4

(b) Find the total length of the curve

$$\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{a}\right)^{2/3} = 1.$$ 3

(c) Find the asymptotes of the curve $x^3 + y^3 = 3axy$. 3

P.T.O.

5. (a) Find the area of the surface when one arch of the cycloid $x = \theta - \sin\theta$, $y = 1 - \cos\theta$ is revolved about the X-axis. 4

(b) Find the points of inflexion on the curve

$$\left(\theta^2 - 1\right) r = a\theta^2.$$ 2

(c) Assuming the convergence of the integrals, prove that

$$\int_0^\infty e^{-x^4} dx \times \int_0^\infty x^2 e^{-x^4} dx = \frac{\pi}{8\sqrt{2}}.$$ 4

6. (a) If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two straight lines equidistance from the origin, show that $f^4 - g^4 = c\left(bf^2 - ag^2\right)$. 5

(b) Find the nature of the conic represented by the equation $3x^2 + 2xy + 3y^2 - 16x + 20 = 0$ by reducing to its canonical form. 5

7. (a) Prove that, if the two conics $\frac{l_1}{r} = 1 - e_1 \cos\theta$ and $\frac{l_2}{r} = 1 - e_2 \cos(\theta - \alpha)$, touch one another, then $l_1^2\left(1 - e_2^2\right) + l_2^2\left(1 - e_1^2\right) = 2l_1 l_2 \left(1 - e_1 e_2 \cos\alpha\right)$. 5

(b) Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 2x - 4y + 2z + 5 = 0$, $x - 2y + 3z + 1 = 0$ is a great circle. 5